Parameter Structure of Noncausal Gauss Markov Random Fields

B. Fahd, T. A. Mehdji, M. Raffael and K. Vel. Prabu
On a paper of Nikhil Balram and José M. F. Moura

Motivation

→ Consider a dependent phenomena described by an elliptic equation (e.g. 2D Image)
→ GMRFs on finite Lattice may result from sampling spatially these phenomena
→ Specify a valid parameter space
→ Estimate the "dependency" parameters

Problem Statement

Markov Random Fields Property:
p(x_r | x_r, for r ≠ s) = p(x_r | N_s)
→ N_s is the neighboring System of the random object X.
→ Relationship between each lattice site and its neighboring system → Parameters describing these relationships.

GMRFs:
- Noncausal Model
- Exponent of the joint probability density function

U(X) = \frac{1}{2\alpha^2}X^TA^2

→ Examine the structure of the Potential matrix A from the pdf of the Noncausal GMRF to specify the valid parameter space.
→ Application: Image Processing, Physical Oceanography ...

Eigenstructure of the Potential Matrix for First-Order Dirichlet Fields

\[ A = \begin{bmatrix} B & C & 0 \\ C & B & C \\ \vdots & \vdots & \vdots \\ 1 & \cdots & 1 \end{bmatrix} \]

\[ B = \begin{bmatrix} 1 & -\beta_{h_1} & 0 \\ -\beta_{h_1} & 1 & -\beta_{h_1} \\ \vdots & \vdots & \vdots \\ 0 & \cdots & 1 \end{bmatrix} \]

Condition:

Positive scaling constant: \sigma^2 > 0
Positive eigenvalues of A: \lambda_{min}(A) > 0

The valid parameter space would be,

\[ |\beta_{v_1}| \cos \frac{\pi}{N+1} + |\beta_{h_1}| \cos \frac{\pi}{M+1} < \frac{1}{2} \]

And this condition corresponds to a region in (\beta_{v_1}, \beta_{h_1}) space bounded by four lines which intersect with the \beta_{v_1} and \beta_{h_1} axes.

Using the parameter structure as derived above on the field, parameters are estimated in a simpler and faster manner by the Maximum Likelihood algorithm.

Conclusion

Non Causal field representation provides better results for 2D spatial operations (than the causal representation).

Estimation of the Parameter Space